



# Cambridge International AS & A Level

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**MATHEMATICS**

**9709/23**

Paper 2 Pure Mathematics 2

**May/June 2021**

**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

- 1 (a) Solve the equation  $\ln(2 + x) - \ln x = 2 \ln 3$ . [3]

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- (b) Hence solve the equation  $\ln(2 + \cot y) - \ln(\cot y) = 2 \ln 3$  for  $0 < y < \frac{1}{2}\pi$ . Give your answer correct to 4 significant figures. [2]

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2 The solutions of the equation  $5|x| = 5 - 2x$  are  $x = a$  and  $x = b$ , where  $a < b$ .

Find the value of  $|3a - 1| + |7b - 1|$ .

[5]

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- 4 (a) Find the exact value of  $\int_0^2 6e^{2x+1} dx$ . [3]

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- (b) Find  $\int (\tan^2 x + 4 \sin^2 2x) dx$ . [5]

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- 5 (a) Find the quotient when  $x^4 - 32x + 55$  is divided by  $(x - 2)^2$  and show that the remainder is 7. [3]

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(b) Factorise  $x^4 - 32x + 48$ .

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(c) Hence solve the equation  $e^{-12y} - 32e^{-3y} + 48 = 0$ , giving your answer in an exact form.

[2]

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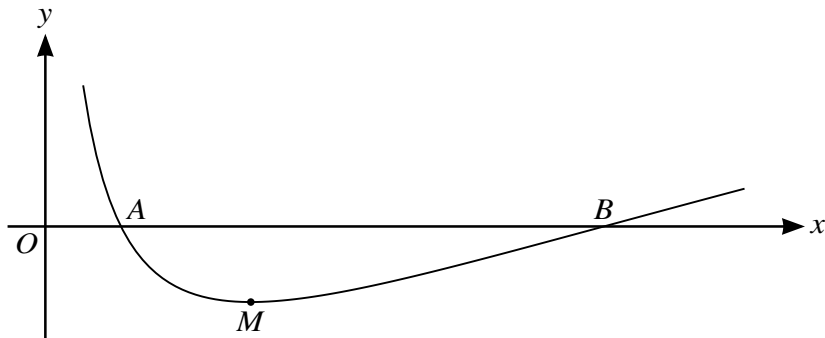
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The diagram shows the curve with equation

$$y = (\ln x)^2 - 2 \ln x.$$

The curve crosses the  $x$ -axis at the points  $A$  and  $B$ , and has a minimum point  $M$ .

- (a) Find the exact value of the gradient of the curve at each of the points  $A$  and  $B$ . [6]

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(b) Find the exact  $x$ -coordinate of  $M$ . [2]

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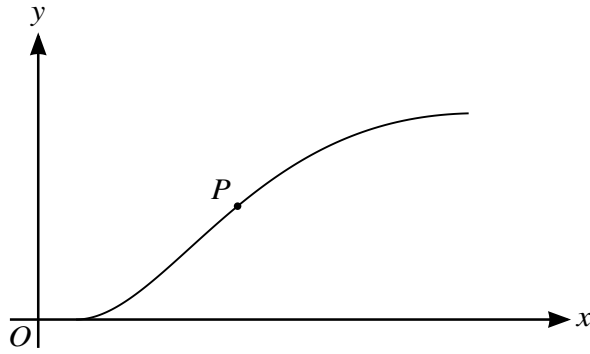
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The diagram shows the curve with parametric equations

$$x = 4t + e^{2t}, \quad y = 6t \sin 2t,$$

for  $0 \leq t \leq 1$ . The point  $P$  on the curve has parameter  $p$  and  $y$ -coordinate 3.

- (a) Show that  $p = \frac{1}{2 \sin 2p}$ . [1]

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- (b) Show by calculation that the value of  $p$  lies between 0.5 and 0.6. [2]

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- (c) Use an iterative formula, based on the equation in part (a), to find the value of  $p$  correct to 3 significant figures. Use an initial value of 0.55 and give the result of each iteration to 5 significant figures. [3]

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(d) Find the gradient of the curve at  $P$ . [5]

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**Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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